"QED – Matter, Light and the Void"

Scientific subject and topic:
Physical properties of light

Title / year:
"QED – Matter, Light and the Void" (2005)

Movie producer:
Scienc*e*motion

Director:
Stefan Heusler

Website of movie:
http://www.sciencemotion.de/

Description of movie:
In the first part of the DVD, the properties of light are shown in a puppet animation movie (30 Min.). Prof. Ethereal and his colleague Nick perform experiments about the physical properties of light and try to explain their results by using models. Not all of their explanations are complete, and not all of their ideas lead to correct conclusions. But their discussions and experiments impart methods of scientific research in a humorous way: A scientist should not be satisfied with just one theory and a corresponding experiment but should try to refine his methods of understanding nature, in this case with the final goal to comprehend the fascinating properties of light better and better.

In the second part of the movie, all the models and experiments are explained on a scientific level using mathematical formulas. In this part, facts of modern research are presented, culminating finally in the theory of quantum electrodynamics (QED). The level of the scenes (about 30) varies between high-school and university level, depending on the difficulty of the specific topic related to the question "What is light?"

Link to Trailer Site:
http://www.sciencemotion.de/

Buy DVD:
Order the DVD for EUR 20.00 plus shipping charge on the website
http://www.sciencemotion.de/

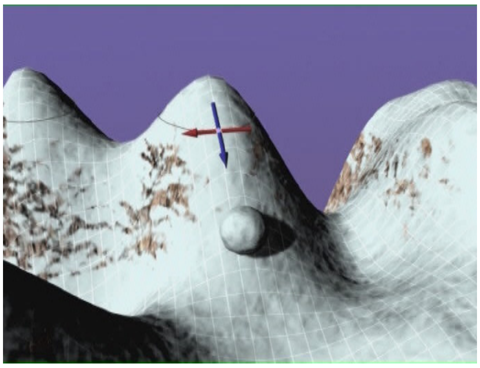
Title of scene:
Introduction to Vector Fields

Video clip or still:
Chapter 3a, Technical Part

Author:
Stefan Heusler, Annette Lorke

Scientific keywords:
classical vector fields, curl field, gradient field

Description of scene:



We are hiking in a snow-covered mountain range. On our contour map of the mountain landscape, the direction of steepest gradient (blue arrows) is shown which defines the gradient field. Along the contour lines red arrows indicate the direction in which the gradient is zero. All red arrows together make up the curl field. At any point, these gradient and curl fields are perpendicular to each other.

After the visual introduction, the fundamental mathematical properties of gradient and curl fields are discussed. Finally, topological curl fields are mentioned, which occur when the space itself is curved.

Author: Stefan Heusler, Annette Lorke
E-mail: sciencemotion@web.de

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Film Studio: Science*e*motion, www.sciencemotion.de

Basic level

While walking in the mountains you can easily understand the most important properties of vector fields. For example, if you are tired and you don't want to climb the mountain, you may search for a path which does not change the altitude. This path runs along the *contour line* on which you are situated. If you want to climb to the top of the mountain as quickly as possible, you should choose the direction of the *steepest gradient*.

If you want to indicate these two directions at the spot where you are standing, you need two arrows. Take two wooden sticks and lay one on top of the other, one pointing the contour line and the other stick perpendicular to the first pointing to the steepest gradient. Imagine you had plastered the whole mountain with pairs of wooden sticks. At any point both sticks have to be perpendicular to each other

The first thing you can observe is that these two sticks are always perpendicular, meaning that the angle between the two sticks is. Imagine that you have laid both sticks at any point of the mountain. With all the sticks you had laid into the direction of the contour line and of the steepest gradient you had created two *vector fields:*

1. The curl field, which winds around all the contour lines;
2. The gradient field, which starts at the top of the mountain and ends in the valley.

Only these two types of vector fields exist – curl fields and gradient fields.

Vector fields occur in many different applications, for example in weather forecasts. Can you think of other examples?

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Advanced level

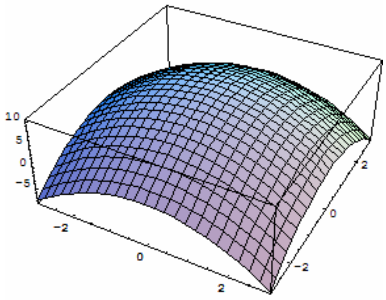
A mountain can be described mathematically by a height function Φ(x, y) which indicates the height of the mountain at the position (x, y). Let's discuss a very simple "mathematical" mountain, with 10 as the maximum height, defined by:

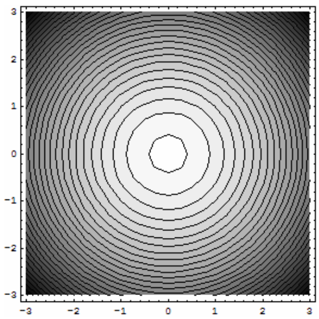
$$\phi(x, y) = 10 - (x^2 + y^2)$$

Furthermore we want to draw a contour map and need the contour lines which are defined by the equation:

$$\phi(x, y) = \text{const.}$$

Let's have a look at this mountain in the range (-3, 3) of the x- and y-coordinates (left picture) and at the contour map (right picture).



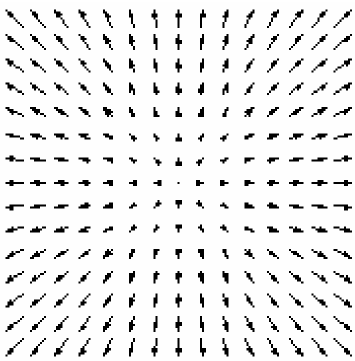


At the origin x = 0, y = 0 the height of the mountain is given by Φ(0, 0) = 10. The contour map shows contour lines with equal height differences between each contour line. The contour lines become denser at the outskirts of the picture because this mountain becomes steeper the more one moves away from its top Φ(0, 0).

The gradient field is a vector field, which consists of two-dimensional vectors.

In the x-coordinate of the vector, we display the slope of the mountain in x-direction, dΦ(x, y)/dx = -2 x.

In the y-coordinate of the vector, we display the slope of the mountain in y-direction, dΦ(x, y)/dy = - 2 y.

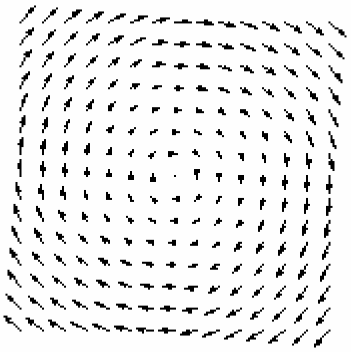


Thus, at any point of the mountain, the gradient vector is given by (-2 x, - 2 y). The result is shown in the figure.

The curl field winds along the contour lines. At any point of the mountain the curl field is perpendicular to the gradient field. It can be calculated through the gradient field by "reversing" the x and y components: (x, y)→ (-y, x). This vector field is orthogonal to the gradient field (- 2 x, - 2 y) because the scalar product is zero.

$$(-2x, -2y) * (-y, x) = 2xy - 2xy = 0$$

This is also valid if you scale a curl field (- y, x) with any constant number c because (- c*y, c*x) is still perpendicular to the gradient field.
For example, for c = 2 the curl field (-2 y, 2 x) is shown here.



These two types of vector fields occur also for more complicated mountain landscapes.

Website about classical vector fields:
http://en.wikipedia.org/wiki/Vector_field

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Scientific level

The classification of vector fields and their relation to topology are described in mathematical terms by the de Rham cohomology group whose basic ideas are presented in the following. The model of our mountain landscape is the trivial case of two dimensions (x, y) in a real, flat space. "Flat space" means that the curvature is zero and the distance dis(x, y) between the origin (0, 0) and the point (x, y) is Euclidian, given by:

$$\mathrm{dis}(x, y) = \sqrt{x^2 + y^2}$$

Let's call this space M. In this space M, all possible differentiable scalar functions Φ(x, y) create the space of all possible mountain landscapes, defined as $\Lambda^1(M)$. Then, we define the space of all possible vector fields, and call it $\Lambda^2(M)$. A general vector field has the form:

$$(f_x(x, y), f_y(x, y))$$

The obvious question is: Can *all* these vector fields be deduced from the space of all mountains $\Lambda^1(M)$? The answer is: Yes, if the space M has the flat metric and hasn't got any holes, which is the case in our example. We can define the differential operator d which acts on Φ(x, y) as:

$$d\phi[x, y] = \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy = \left(\frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y}\right)$$

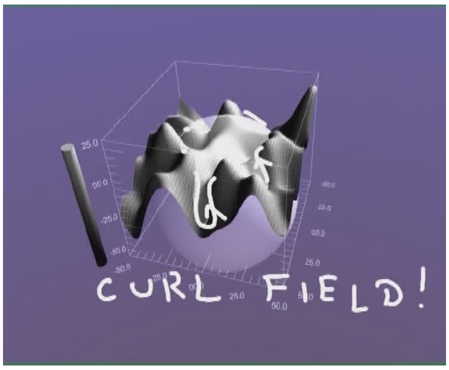
This defines a mapping from $\Lambda^1(M)$ into $\Lambda^2(M)$, simply by linking each mountain to its corresponding gradient field. Within $\Lambda^2(M)$ we may now form the orthogonal complement to the image dΦ(x, y). Obviously, this is the curl field related to the contour lines of each mountain Φ(x, y), given by:

$$-\frac{\partial \phi}{\partial y} dx + \frac{\partial \phi}{\partial x} dy = \left(-\frac{\partial \phi}{\partial y}, \frac{\partial \phi}{\partial x}\right)$$

This completes the classification of all vector fields in $\Lambda^2(M)$ for the flat, two-dimensional space. Of course, a given vector field (f_x(x, y), f_y(x, y)) can consist of a linear combination of a gradient and a curl field which are related to two different mountains Φ_1, Φ_2.Therefore, the gradient- and curl field components of a general vector field (f_x(x, y), f_y(x, y)) are not necessarily orthogonal to each other.

Besides the gradient field dΦ(x, y) and its orthogonal complement, there exists one more class of vector fields: Imagine the two-dimensional space M to be the surface of a sphere – as shown in the movie. The sphere's surface has a positive curvature is positive. You can easily construct a curl field winding around the sphere. This field occurs even for the trivial mountain function Φ = const. This curl field cannot be constructed as the orthogonal complement of dΦ = 0. It is the simplest example of a *topological curl field.*

Now we have described all three types of existing vector fields: (1) gradient fields, (2) curl fields which are perpendicular to a gradient field, (3) topological curl fields.



By the way, the orthogonal complement of a topological curl field is – if it exists – once more a topological curl field.

Website about the Cohomology group:
http://en.wikipedia.org/wiki/De_Rham_cohomology