“QED – Matter, Light and the Void”

Scientific subject and topic:
Physical properties of light

Title / year:
“QED – Matter, Light and the Void” (2005)

Movie producer:
Scienc*e*motion

Director:
Stefan Heusler

Website of movie:
http://www.sciencemotion.de/

Description of movie:
In the first part of the DVD, the properties of light are shown in a puppet animation movie (30 Min.). Prof. Ethereal and his colleague Nick perform experiments about the physical properties of light and try to explain their results by using models. Not all of their explanations are complete, and not all of their ideas lead to correct conclusions. But their discussions and experiments impart methods of scientific research in a humorous way: A scientist should not be satisfied with just one theory and a corresponding experiment but should try to refine his methods of understanding nature, in this case with the final goal to comprehend the fascinating properties of light better and better.

In the second part of the movie, all the models and experiments are explained on a scientific level using mathematical formulas. In this part, facts of modern research are presented, culminating finally in the theory of quantum electrodynamics (QED). The level of the scenes (about 30) varies between high-school and university level, depending on the difficulty of the specific topic related to the question “What is light?”

Link to Trailer Site:
http://www.sciencemotion.de/

Buy DVD:
Order the DVD for EUR 20.00 plus shipping charge on the website
http://www.sciencemotion.de/

Artistic Part, Chapter 4

Title of scene:
The atom

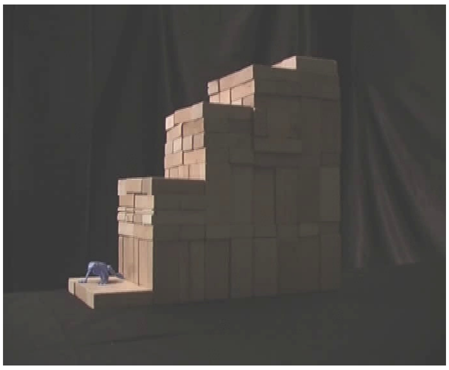
Video clip or still:
Chapter 4, Artistic Part

Time interval:
starting at 2:35

Author:
Stefan Heusler, Annette Lorke

Scientific keywords:
energy levels in the atom, Bohr's atomic model, Planck's constant, positrons

Description of scene:



Prof. Ethereal and his assistant Nick burn different kinds of elements (e.g. calcium) and find that the colour of the burning flame depends on the type of the atom. In order to explain their observations they construct a simple *model*: a staircase on which a frog is sitting on a step. Each step of this staircase represents a different energy level in an atom whereas the frog stands for an electron of this atom. The frog can jump from step to step on the staircase. To be able to jump to a higher level, it must absorb a photon and therefore gains energy. If it jumps to a lower level, it looses energy and emits a photon.
A jump is only possible if the energy difference of two steps exactly corresponds to the energy of the photon. Therefore, the energy of the photon and its colour are unique for each jump. Since each atom has a characteristic staircase, it has a unique spectrum of colours which are emitted by the atom. This fact is used for identifying the type of atom in spectral analysis.

Prof. Ethereal and Nick compare the staircase model to Bohr's atomic model. In Bohr's model it is misleading to speak about one fixed orbit for each electron because the electron does not follow one specific orbit but shows characteristics of a standing wave. Then professor narrates some of the history of atom physics. In 1928 Paul Dirac combined the theories of special relativity and quantum mechanics. Dirac was able to calculate the "energy staircase" (the spectrum) of the hydrogen atom with very high accuracy. Moreover, as a result of his calculations he predicted the existence of antimatter – the positron.

Author: Stefan Heusler, Annette Lorke
E-mail: sciencemotion@web.de

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Basic level

The possible existence of the "inseparable substance" has been fascinating mankind for a long time. The ancient Greeks thought that an element consists of very small pieces which cannot be separated anymore and called this piece "atom" (inseparable). However, nowadays we know that an atom can be further separated into the negatively charged electrons and the positively charged atomic nuclei consisting of protons and neutrons. The simplest possible example is the hydrogen atom, which possesses only one electron, one proton and no neutron. To get an idea about the size of the hydrogen atom, take one millimetre and line up 10 million hydrogen atoms in this millimetre.

Is it possible to see an atom, for example through a microscope? No, it is impossible to see an atom directly. There are only indirect methods to observe the particles of an atom. For example an electron can be detected by the light that it emits. In this case the electron jumps from a higher to a lower step on its *energy staircase* and sends out a photon.

For the hydrogen atom the complete energy levels of the electron (spectrum of the atom) can be precisely measured and calculated. Many scientists had been working on these measurements and calculations before the findings became the cornerstone in the development of quantum mechanics in the 20th century. It is important to note that only for the hydrogen atom it is possible to carry out exact measurements and calculations which correspond to each other. For all other atoms, e.g. carbon, the mathematical equations are still too complicated to be solved with the same accuracy.

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Advanced level

Our model of the energy staircase describes the so-called quantization of energy levels for the electrons in an atom. If there wasn't any quantization the electrons could take on continually any kind of energy which could be best depicted with an inclined plane rather than a staircase.

By analysing the emitted radiation of all the atoms we make the observation that each atom possesses a characteristic spectrum. However, we do not know the reason for it. The best we can do is to find a *common principle* behind the different specific spectra.

The energy staircase of the atom is based on a very fundamental common principle has been discovered. If we postulate that the position x and the momentum p of the electron do not commute and their commutator[1] is proportional to Planck's constant h, we find that the energy levels of the electrons in the atom are discrete.

Mathematically speaking:

$$x\,p - p\,x = i\,\frac{h}{2\pi}, \qquad i = \sqrt{-1}$$

The postulate is the common principle which successfully describes the behaviour of the electrons in the atom, $h = 6.6 \cdot 10^{-34}$ Js is Planck's constant. The number i is the so-called complex unit. It is defined as i^2 equals minus one:

$$i^2 = \left(\sqrt{-1}\right)^2 = -1$$

The fact that position and momentum do not commutate is valid not only for electrons but for *any* kind of matter. It is the key to quantum mechanics.

Paul Dirac tried to combine the theory of special relativity and quantum mechanics. The relativistic energy of a free particle is[2]:

$$E^2 = p^2 c^2 + m_0^2 c^4$$

A free electron with the rest mass m, the momentum p and without any exterior potential (e.g. Coulomb potential) has the energy E. This equation only contains one constant of nature, the speed of light $c \approx 300\,000$ km/s. The constant $h = 6.6 \cdot 10^{-34}$ Js is introduced into the theory through the commutator relation $x*p - p*x = i*h/(2\pi)$.

Dirac tried to find an equation which describes the electron in such a way that both the non-commutativity of position and momentum and the fact that the speed of light is the maximum possible speed are respected. Obviously, his equation must contain both the speed of light c and Planck's constant h. Dirac's aim was to reproduce exactly the energy staircase of the electron in the hydrogen atom.
In 1928, Dirac succeeded and formulated his famous equation, nowadays called Dirac's equation. The energy staircase he calculated in those days reproduced the data for the hydrogen atom which had been experimentally found until then.
Moreover his equation made a very far-reaching prediction:
If the energy step E_{step} is a solution to his equation, $-E_{step}$ is also a solution. This is

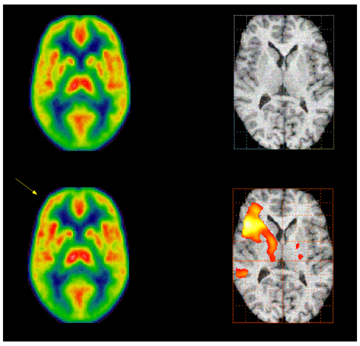
[1] The commutator is explained in more detail in chapter 4b on the DVD.
[2] This equation is derived in chapter 1f on the DVD.

already obvious from the relativistic equation for the energy, which is quadratic. The physical interpretation of the positive energy values ($E_{step\ 1}$, $E_{step\ 2}$, $E_{step\ 3}$,...) is the following: They form the steps of the energy staircase, on which the electron can jump up and down by emitting or absorbing photons.
How can we interpret the steps with negative energy ($-E_{step\ 1}$, $-E_{step\ 2}$, $-E_{step\ 3}$, ...)? Even for Paul Dirac it wasn't easy to find the correct answer. After some struggle he predicted that there had to be a particle which had exactly the same mass as the electron but with an opposite charge. This particle is nowadays called the *positron*, the anti-particle of the electron. However, both matter and anti-matter have positive energy.

If an electron and a positron collide, they decay into two photons (denoted as $\gamma + \gamma$). The energy of the photons is the sum of the rest mass (511 keV + 511 keV) and the kinetic energy of both the electron and the positron. If the electron and the positron are almost at rest, each of the two photons, created in the reaction, have an opposite momentum, the energy of about E = 511 keV and the frequency ν=E/h.

$$e^+ + e^- \rightarrow \gamma + \gamma$$

Meanwhile this reaction is used as a medical diagnosis method for depicting the glucose density in the brain. The positrons necessary for the reaction are created by injecting radioactive glucose into the blood. This method is called Positron-Emission-Tomography (PET).



PET-image of the brain (from http://nru.dk/research/illustrations/), red colour indicates high density of decaying electron/positron pairs in radioactive glucose (β^+-decay)

Website about Positron Emission Tomography
http://en.wikipedia.org/wiki/Positron_emission_tomography

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Scientific level

We want to demonstrate that the angular momentum algebra (Chapter 4d) is a direct consequence of the fundamental commutator relation x*p - p*x = i*h/(2π). For this purpose, let us introduce a three-dimensional coordinate system (x, y, z). The angular momentum is defined in classical mechanics as

$$J_x = y\,p_z - z\,p_y, \quad \text{and cyclic}$$

The equations for L_y and L_z can be obtained by a cyclic rotation x→ y→ z→ x→ y... in the above equation:

$$J_y = z\,p_x - x\,p_z$$
$$J_z = x\,p_y - y\,p_x$$

For example, a ball with the mass m which rotates around the z-axis with the angular frequency w is described by the coordinates (x, y) and the momentum (p_x, p_y) as

$$x = r\,\mathrm{Cos}[w\,t], \quad p_x = m\frac{d}{dt}x = -m\,w\,r\,\mathrm{Sin}[w\,t]$$
$$y = r\,\mathrm{Sin}[w\,t], \quad p_y = m\frac{d}{dt}y = m\,w\,r\,\mathrm{Cos}[w\,t]$$

The angular momentum of this ball is given by

$$J_z = x\,p_y - y\,p_x = m\,w\,r^2$$

In classical mechanics, any value for the angular frequency w is allowed. However, if we introduce the commutator relation $x*p_x – p_x*x$ = i*h/(2π) (and cyclic), we can *derive* the commutator relations between J_x, J_y and J_z

$$[J_x, J_y] = i\frac{h}{2\pi}J_z$$

As shown in chapter (4 d) of the DVD, the solution of this algebra is a staircase with any number of steps. The step height is h/(2 π). Physically, this means that the angular momentum can change its value only in jumps of the step height h/(2 π). It impossible to create an angular momentum which is smaller than h/(2 π).

In chapter (4d), we show the angular momentum algebra in a different basis with different operators (the so-called creation and annihilation operators). The relation to J_x, J_y und J_z is given by:

$$J^+ = \frac{1}{\sqrt{2}}(J_x + i\,J_y), \quad J^- = \frac{1}{\sqrt{2}}(J_x - i\,J_y), \quad J_z$$

In this basis, the same algebra reads:

$$[J_z, J^+] = \frac{h}{2\pi}J^+, \quad [J_z, J^-] = -\frac{h}{2\pi}J^-,$$
$$[J^+, J^-] = \frac{h}{2\pi}J_z$$

This quantization of angular momentum is not a new postulate, but follows from the fundamental commutator relation x*p - p*x = i*h/(2π).

Websites about the hydrogen
http://en.wikipedia.org/wiki/Hydrogen
http://en.wikipedia.org/wiki/Hydrogen_spectral_series