"QED – Matter, Light and the Void"

Scientific subject and topic:
Physical properties of light

Title / year:
"QED – Matter, Light and the Void" (2005)

Movie producer:
Science*e*motion

Director:
Stefan Heusler

Website of movie:
http://www.sciencemotion.de/

Description of movie:
In the first part of the DVD, the properties of light are shown in a puppet animation movie (30 Min.). Prof. Ethereal and his colleague Nick perform experiments about the physical properties of light and try to explain their results by using models. Not all of their explanations are complete, and not all of their ideas lead to correct conclusions. But their discussions and experiments impart methods of scientific research in a humorous way: A scientist should not be satisfied with just one theory and a corresponding experiment but should try to refine his methods of understanding nature, in this case with the final goal to comprehend the fascinating properties of light better and better.

In the second part of the movie, all the models and experiments are explained on a scientific level using mathematical formulas. In this part, facts of modern research are presented, culminating finally in the theory of quantum electrodynamics (QED). The level of the scenes (about 30) varies between high-school and university level, depending on the difficulty of the specific topic related to the question "What is light?"

Link to Trailer Site:
http://www.sciencemotion.de/

Buy DVD:
Order the DVD for EUR 20.00 plus shipping charge on the website
http://www.sciencemotion.de/

Technical Part, Chapter 4b

Title of scene:
The commutator

Video clip or still:
Chapter 4b, Technical Part

Author:
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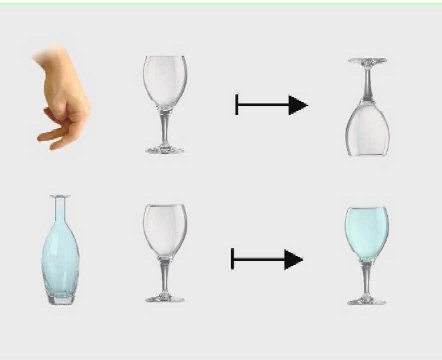
Scientific keywords:
commutator, operators, eigenstate, eigenvalue

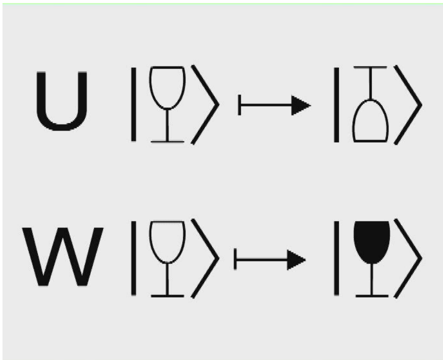
Description of scene:

We show a simple example for the so-called commutator. For many actions their order is important. Our example demonstrates two actions with two operators:

1) Applying the operator U means turning a glass upside down.
2) Applying the operator W means pouring water from a bottle.

At first, these operations are illustrated with pictures. Then they are described by using mathematical symbols.





Finally both operators are represented in two different 3 times 3 matrices. These two matrices can be multiplied in two different ways (U*W and W*U). If the result of multiplying the matrices is not the same, the order of the applications is important. This is described by the difference U*W – W*U which defines the commutator. If the sequence of the operations does not matter, the commutator is zero.

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Movie clip: Chapter 4b, Technical Part
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Film Studio: Science*motion*, www.sciencemotion.de

Basic level

The sequence of actions is important. Before you leave a room, you should open the door. Before you can choose the TV programme on the remote control, you must switch on the TV. Therefore it should not be astonishing that the sequence of applications is also important in physics and mathematics. Let's start with a very simple mathematical operation: If you have an empty bag and you put 5 apples into it and add 4 more apples you finally have 9 apples in your bag. The same happens if you put 4 apples into the empty bag and then the five. Both ways have the same result which means in the mathematical language that additions commutate.

Fascinating mathematical structures will arise if two operations do not commutate. In the movie, we show the example of an empty glass, a bottle of water and two operations: The first operation turns the glass upside down (described by the operator U) and the second operation pours water from the bottle (described by the operator W). You can try this for yourself and find out that the sequence of the operations is important or in other words that the operations do not commutate. You may also invent other operations and find out whether they commutate or not.

If you want to apply an operation you need an initial state from which you can start your observations of the operator's effect. In our example the empty glass is the initial state. If you apply an operator to the glass, it can change some of the glass's properties. The *initial state* before the operation is changed to the *final state* after the operation.

As there are different operators which are applied to our glass there are different final states of the glass. Which is the full set of all possible final states starting with the empty glass as the initial state, if you apply any amount of operators U and W to it? Whatever you do, the final state will always be one of the following three states:

1) Glass upright, empty
2) Glass turned upside down, empty
3) Glass upright, filled

The state "glass turned upside down, filled" does not exist as you can see in the experiment and therefore does not appear in the mathematical formula.

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Advanced level

The operations U and W can be represented as two 3 times 3 matrices because there are three final states possible. In general, the application of an operator to an initial state leads to a different state. However, sometimes the operator does not change the initial state. This important case is called *eigenstate* of the operator. For example, the state "glass upright, filled" is an eigenstate of W, since

W |Glass upright, filled> = |Glass upright, filled>

But it is not an eigenstate of U, since

U |Glass upright, filled> = |Glass turned upside down, empty>

Applying an operator to an eigenstate can in some sense be regarded as the "measurement" of a property of this eigenstate. It depends on the operator which sort of property you can measure.

In the following, we want to find all the eigenstates of the operator W which means that we want to solve the equation:

W |initial state> = λ * |initial state> (λ is the result of the measurement and called the *eigenvalue*).

Of course, we already know with the definition of W that one solution is:
|eigenstate 1> = |glass upright, filled> (with λ = 1).

This is not the only possible solution, since:
|eigenstate 2> = |glass turned upside down, empty> (with λ = 1).

But there is one more solution, which is less obvious. It is given by:
|eigenstate 3> = - |glass upright, empty> + |glass upright, filled> (with λ = 0)

We can easily check that this is an eigenstate, because:
W |eigenstate 3> = - |glass upright, filled> + |glass upright, filled> = 0
= 0 * |eigenstate 3>

In general a 3 times 3 matrix cannot have more than three eigenstates.

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Scientific level

Given the 3 times 3 matrices U and W, what are their eigenvectors and eigenvalues? To answer this question it is necessary to diagonalize both matrices. We choose W as an example:

$$W v_1 = \lambda_1 v_1, \quad v_1 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$W = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \qquad W v_2 = \lambda_2 v_2, \quad v_2 = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$$

$$W v_3 = \lambda_3 v_3, \quad v_3 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

The eigenvalues $(\lambda_1, \lambda_2, \lambda_3) = (1, 1, 0)$ are nothing but the diagonalized matrix $W_{diag.}$ in the basis of the eigenstates.

$$W_{diag.} = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

At first sight these formal equations tend to lack meaning. If they are interpreted with a proper example such as our glass-bottle example it is easier to understand them. Nevertheless, our interpretation of the third eigenvector v_3 (with vanishing eigenvalue, $\lambda_3 = 0$) doesn't seem to have much in common with reality:

v_3= - |Glass upright, empty> + |Glass upright, filled>
W v_3 = - |Glass upright, filled> + |Glass upright, filled> = 0

In real life it is impossible to create the state "minus glass upright, filled".
Paul Dirac stumbled over states with negative energy in his famous equation. For his equation about the electron he found formal solutions which did not seem to fit to a corresponding experiment. The states with negative energy have finally been interpreted correctly as anti-matter with positive energy. Therefore one should not jump hastily to the conclusion that strange mathematical equations do not have any relation to nature. Mathematics tends to be very useful as a pathfinder to new discoveries. Of course, our glass-bottle model is much too simple for such discoveries.

Websites about the commutator and its significance
http://en.wikipedia.org/wiki/Commutator
http://en.wikipedia.org/wiki/Uncertainty_relation