"QED – Matter, Light and the Void"

Scientific subject and topic:
Physical properties of light

Title / year:
"QED – Matter, Light and the Void" (2005)

Movie producer:
Science*e*motion

Director:
Stefan Heusler

Website of movie:
http://www.sciencemotion.de/

Description of movie:
In the first part of the DVD, the properties of light are shown in a puppet animation movie (30 Min.). Prof. Ethereal and his colleague Nick perform experiments about the physical properties of light and try to explain their results by using models. Not all of their explanations are complete, and not all of their ideas lead to correct conclusions. But their discussions and experiments impart methods of scientific research in a humorous way: A scientist should not be satisfied with just one theory and a corresponding experiment but should try to refine his methods of understanding nature, in this case with the final goal to comprehend the fascinating properties of light better and better.

In the second part of the movie, all the models and experiments are explained on a scientific level using mathematical formulas. In this part, facts of modern research are presented, culminating finally in the theory of quantum electrodynamics (QED). The level of the scenes (about 30) varies between high-school and university level, depending on the difficulty of the specific topic related to the question "What is light?"

Link to Trailer Site:
http://www.sciencemotion.de/

Buy DVD:
Order the DVD for EUR 20.00 plus shipping charge on the website
http://www.sciencemotion.de/

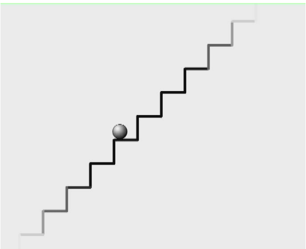
Title of scene:
The angular momentum algebra

Video clip or still:
Chapter 4d, Technical Part

Author:
Stefan Heusler, Annette Lorke

Scientific keywords:
quantization of angular momentum, electron spin, periodic table of elements, Pauli principle

Description of scene:



We show a staircase and a ball on one of its steps. Three *operations* are introduced:

1) The ball jumps one step up.
2) The ball jumps one step down.
3) The ball stays where it is and gives the name of the step.

These three operations can be applied to a ball on any staircase with N steps. For an odd number of steps (N = 2l + 1, l integer), this model turns out to be related to normal *rotations*. With *normal* we mean that after a rotation of 360° degree, we come back to the starting point. In quantum mechanics there exits another, quite peculiar rotation in the so called the “spin space”. In the spin space we come back to the starting point only after a rotation of 720° degree. This spin space can also be described by our model. In this case the number of steps on the staircase is even (N = 2 l).

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Basic level

You have probably gone up on staircases many times without thinking too much about the number of steps, whether it's an even or odd number. Maybe it will astonish you that the difference between these two staircases is enormous in mathematical physics. To explain this, imagine yourself standing on one of the steps of the staircase. Obviously, you can do three different things: You can go up one step, go down one step, or you can stay where you are. Now we give each step a number, so you can identify the step you are standing on, e.g. step number 7.
A normal way to number steps would be to start with the lowest step and to number the steps such as "step one, step two, step three, etc". But our staircase is not a normal one. It is a magic staircase which exists somewhere in the universe where you cannot differentiate between its top and bottom. How can you number the steps of this peculiar staircase? Imagine that you stand somewhere on this stairs. The first thing you should do is to go as many steps in one direction until you reach the end of the stairs. Starting from this end, count the number of steps while going in the other direction. When you reach the other end of the staircase, turn around, walk back and count the steps in reverse order. For a staircase with 5 steps, each step is counted in these two different manners, which can be written down as:

$$1 \to 2 \to 3 \to 4 \to 5$$
$$5 \to 4 \to 3 \to 2 \to 1$$

Look! There is one step which keeps its number in both counting manners. It's the number for the step in the *middle* of the staircase. Obviously, this step is somehow special. We will *renumber* the steps by assigning the number zero to the step in the middle. In our example we subtract 3 from each step number.

$$1-3 \to 2-3 \to 3-3 \to 4-3 \to 5-3$$
$$5-3 \to 4-3 \to 3-3 \to 2-3 \to 1-3$$

Then, we have the following step numbers:

$$-2 \to -1 \to 0 \to 1 \to 2$$
$$2 \to 1 \to 0 \to -1 \to -2$$

The counting starts in the *middle* of the staircase at step number zero.
This kind of numbering is symmetric around the middle step. That's the most natural thing you can do when you don't know where the bottom and the top of the staircase are.

Now it's your turn to find a ***symmetric*** numbering for a staircase with ***two*** steps. If you succeed, you will have a good model to describe the *electron spin*.

As above the first thing you do is to count the steps in two different directions. The result is:

$$1 \to 2$$
$$2 \to 1$$

We have a little problem: There is no middle step. Thus we cannot assign zero to a step from which we can start counting in both directions in order to get a symmetric numbering. Nevertheless we want the staircase to be symmetric. We could try the following numbers:

$$-1 \to +1$$
$$+1 \to -1$$

Indeed, the counting is symmetric. However, a new problem has occurred. The height has changed its value from 1 to the value 1-(-1) = 2.

This is not the same stairs we have started with. But we still need the stairs on which each step has the height one. We obtain such a staircase by dividing the numbers by two. This leads to the step numbers:

$$-\frac{1}{2} \rightarrow +\frac{1}{2}$$
$$+\frac{1}{2} \rightarrow -\frac{1}{2}$$

And this step numbering is used for describing the electron spin. Of course, there is more to the electron spin than meets the eye. More mathematical problems have to be solved to describe the spin in detail. But you have grasped the basic idea behind the spin with the model of the staircase with two steps: The spin can have two states which are described by two half-integer values, either “spin +½” or “spin -½”.

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Advanced level

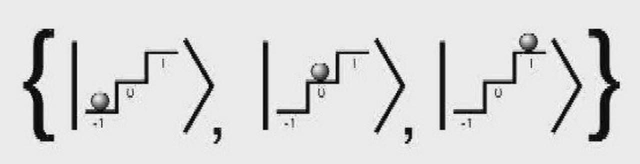
What is the angular momentum? It is the rotation of an object relative to a certain reference point. For example, a spinning top that rotates around its axis has relative to its axis an angular momentum. The faster it rotates, the larger its angular momentum is. For a spinning top the rotation velocity can be increased continuously.

This doesn't apply for an electron in the atom. Here, the rotation velocity cannot be changed continuously: The angular momentum is quantized, meaning that it can only be increased or decreased in portions. The size of these portions is given by Planck's constant $h/(2\pi) = 1.055 \cdot 10^{-34}$ Js.

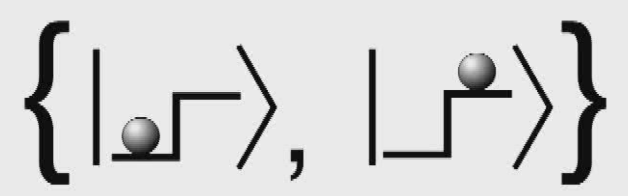
We introduce a staircase as a model for the quantized angular momentum. For example if the angular momentum of an electron in the atom is reduced about one portion $h/(2\pi)$, it corresponds to the ball in our model that jumps one step downwards on the staircase.

The total number of steps is determined by the absolute value of the electron's angular momentum. If the electron has no angular momentum at all, this corresponds to a staircase with only one single step which has the number zero.

If the absolute value of the angular momentum is not zero, the electron can either rotate in positive or negative direction. The simplest possible case in our model is a staircase with three steps: **+1** $h/(2\pi)$, **0** $h/(2\pi)$, **-1** $h/(2\pi)$.



staircase model of the angular momentum with l = 1



staircase model of the electron spin

In general, the staircase which describes the angular momentum has an **odd number of steps**. The total number (2 l + 1) of steps of the "angular momentum staircase" is determined by the maximal possible angular momentum component projected onto a given axis is **l***$h/(2\pi)$.
In general, the staircase which describes the spin has an **even number of steps**. The electron spin, projected onto a given axis, has only two possible states. This corresponds to a staircase with only two steps, with 2*S + 1 = 2 and S = ½.

Electrons both have an angular momentum (with any maximal value l, corresponding to a staircase with any odd number of 2*l +1 steps) and a spin (with the maximal value S = ½, corresponding to a staircase with 2*S + 1 = 2 steps.). To describe the rotation state of the electron in our staircase model, we have to specify the electron's step position both on the angular momentum staircase and on the spin staircase.
If the maximal angular momentum corresponds to l, we have 2(2*l + 1) possibilities of the combination of angular momentum and spin states.

We already can describe the most important properties of the periodic table of elements with this model of the electron's rotation state.

Group→ / ↓Period	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1 H																	2 He
2	3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
3	11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
4	19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
5	37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
6	55 Cs	56 Ba	*	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
7	87 Fr	88 Ra	**	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Uub	113 Uut	114 Uuq	115 Uup	116 Uuh	117 Uus	118 Uuo

The order of the elements corresponds to total number of electrons in the atom: Hydrogen (H) has one electron, Helium (He) has two electrons, Lithium (Li) has three electrons, and so on. Take Silver (Au) with 79 electrons as another example. How are these electrons arranged in the atom? The experimental results might surprise you but these electrons sit on the steps of two different kinds of staircases, which correspond to different portions of the angular momentum (odd number (2 l + 1) of steps) and the spin (two possible steps). Wolfgang Pauli, one of the fathers of quantum mechanics, discovered that two electrons **never sit on the same step**. This is the so-called Pauli-principle.

The electron in the atom can be described by the **quantum numbers (n, l, m, S=½, s)**. Except for the principal quantum number n, which is related to the period n=1, 2, 3, ... in the periodic table of elements, all the other quantum numbers (**l, m)** and **(S=½, s**) are related to the angular momentum staircase and the spin staircase of our model. The quantum numbers describe the rotation state of the electron.

The numbers (**l, m**) describe the angular momentum of the electron. The **total angular momentum quantum number l** corresponds to a staircase with (2*l + 1) steps. The so-called **magnetic quantum number m** describes the step position of the electron on this staircase (m = - l, ..., -1, 0, 1, l).

The numbers (**S=½, s**) describe the electron spin. The **total spin S** of the electron is always S=½, thus the spin staircase always has two steps. The **spin quantum number s** describes the step position of the electron on this two-step staircase and is either s = - ½ or s= ½.

Let's see how our staircase model works with the elements of the first period, n=1. Here, the electron has no angular momentum, l=0. In the hydrogen atom, there is only one electron which has the principal quantum number n=1, the angular momentum quantum numbers (l=0, m=0) and the spin quantum numbers (S=½, s=+½).

Next, consider Helium. In the Helium atom there are two electrons. According to the Pauli principle the two electrons must sit on different steps. However the angular momentum staircase for l=0 has only 2*l + 1 = 1 step. Therefore, each electron must have a different spin: One electron sits on step (s=+½), the other on step (s=-½).

How many states exist with the principal quantum number n=2? To answer this question, you need further information derived from quantum mechanics: In general for a given period n, the total angular momentum can only take on the values l=0, 1, ..., n-1.
For n=2, the total angular momentum can be either l=0 or l=1. For l=0, two spin states are possible.
For l=1, the angular momentum corresponds to a staircase with three steps: m=-1, 0, 1. Again there are two possible spin states on each step of this angular momentum staircase.
In total, we obtain 2(1 + 3) = 8 different quantum numbers for electrons in the period n=2. These eight states correspond to the eight atoms (Li, Be, B, C, N, O, F, Ne).

In each period there is an electron configuration in which all the staircases are either completely filled or completely empty. These atoms are chemically especially stable and are called noble gases. Atoms which are not noble gases have staircases on which some steps are filled, and some steps are empty. To reach a stable electron configuration these atoms tend to react with each other and form molecules with completely filled angular momentum and spin staircases. The simplest example is hydrogen gas H_2.

Websites about the periodic table & quantum chemistry

http://en.wikipedia.org/wiki/Periodic_table
http://en.wikipedia.org/wiki/Quantum_chemistry
http://en.wikipedia.org/wiki/Pauli_exclusion_principle

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Scientific level

The staircase model describes the angular momentum and the spin of the electron very well. However, within this model it is not possible to predict the order in which the steps are occupied.

The elements are arranged in the periodic table of elements according to the number of electrons. The order of filling the electrons' states is determined by the **binding energy** of the valence electrons. The staircase model does not explain the binding energy. For example, the 10 elements from Sc to Zn occur in period 4 in the table of elements *after* the two elements (K, Ca) with n=4 and the angular momentum l=0 although they have the principal quantum number n=3 and the total angular momentum l=2.

In 1925, Wolfgang Pauli formulated his exclusion principle, which is valid for all particles with half-integer spin. It was first formulated as an empirical result without proof. Later on, a mathematical basis came with the **spin-statistics theorem** which states that particles with half-integer spin must have an anti-symmetric wave function and particles without half-integer spin must have a symmetric wave function. Today, these two kinds of particles are called fermions and bosons.

Let's have a look at the consequences of the spin-statistics theorem in a simple example. Consider the wave function of two electrons, described by the variables e1 and e2. According to the spin-statistics theorem, the wave function must be anti-symmetric under exchange of the electrons 1 and 2:

$$\psi(e1, e2) = -\psi(e2, e1)$$

In particular, this means that two electrons which are described by the same set of variables (the same quantum numbers) cannot exist:

$$\psi(e1, e1) = -\psi(e1, e1) \rightarrow \psi = 0$$

Note that for bosons this conclusion cannot be drawn because in this case the wave function is even. Therefore, many bosons with the same quantum numbers can coexist.

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